

Microwave Devices - Homework n.4

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According to HW2, the following parameters are used for the matching analysis:

$$Z_0 = 50 \, \Omega, \quad Z_L = 705 \, \Omega, \quad f_0 = 2365 \, \text{MHz}$$

1 Matching with Lumped Elements at the Central Frequency

The normalized load impedance is

$$z_L = \frac{Z_L}{Z_0} = 14.1. \quad (1)$$

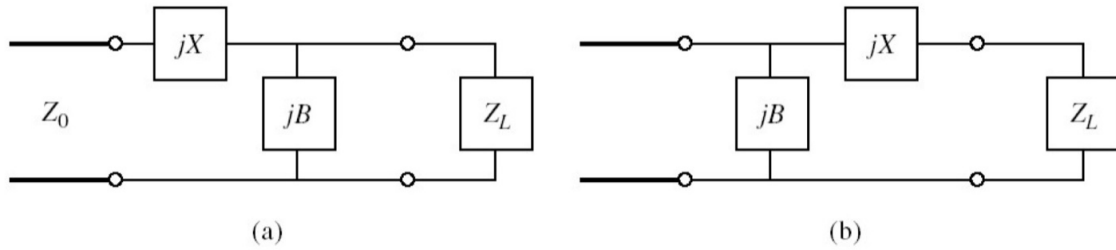


Figure 1: L-section matching networks: (a) configuration for normalized load impedance inside the $1 + jx$ circle; (b) configuration for normalized load impedance outside the $1 + jx$ circle.

Since the normalized load impedance lies outside the $1 + jx$ circle on the impedance Smith chart, the L-section matching network configuration (b) must be adopted. The two possible L-section configurations are shown in Fig. 1; configuration (b), characterized by a shunt reactive element followed by a series reactive element when observed from the generator side, is therefore selected for the present design.

For a purely real load ($X_L = 0$) and for $R_L > Z_0$, the analytical solutions of the system are

$$X = \pm \sqrt{Z_0 (R_L - Z_0)}, \quad (2)$$

$$B = \pm \sqrt{\frac{R_L - Z_0}{Z_0 R_L^2}}. \quad (3)$$

Substituting the numerical values yields

$$X = \pm 180.97 \, \Omega, \quad (4)$$

$$B = \pm 5.13 \times 10^{-3} \, \text{S}. \quad (5)$$

Choosing the low-pass realization, the series reactance is inductive ($X > 0$) and the shunt susceptance is capacitive ($B > 0$). The corresponding lumped element values at the central frequency are

$$L = \frac{X}{\omega_0} \approx 12.18 \text{ nH}, \quad (6)$$

$$C = \frac{B}{\omega_0} \approx 0.345 \text{ pF}, \quad (7)$$

where $\omega_0 = 2\pi f_0$.

At the central frequency, the L-section network matches the load to the transmission line, providing an input impedance equal to Z_0 .

2 Input reflection coefficient versus frequency

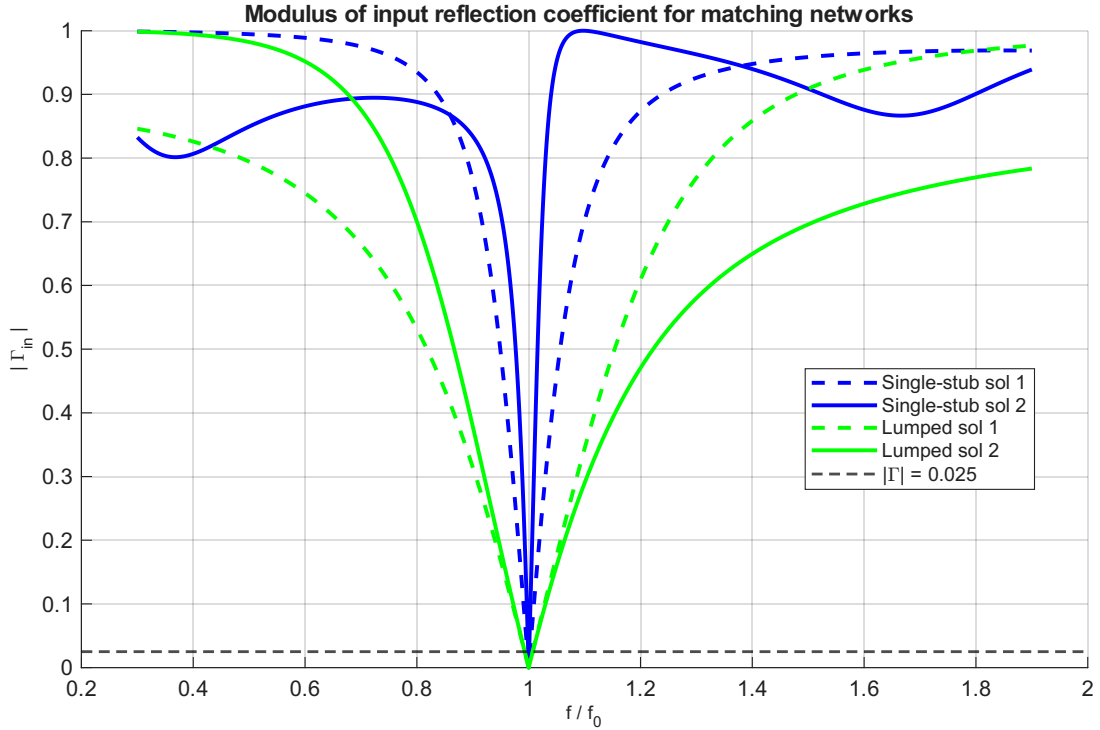


Figure 2: Modulus of the input reflection coefficient for single-stub and lumped matching networks as a function of the normalized frequency. The horizontal line indicates the threshold $|\Gamma_{in}| = 0.025$.

Figure 2 shows the modulus of the input reflection coefficient $|\Gamma_{in}|$ as a function of the normalized frequency f/f_0 in the range $0.3f_0 \leq f \leq 1.9f_0$. In all cases, perfect matching is achieved at the design frequency f_0 , where $|\Gamma_{in}|$ vanishes. However, the frequency behavior away from f_0 highlights a markedly different bandwidth performance, with the lumped matching networks providing a wider frequency interval below the imposed reflection coefficient threshold compared to the single-stub solutions.

3 Bandwidth evaluation

The bandwidth has been numerically evaluated in the frequency range $0.3f_0 \leq f \leq 1.9f_0$ as the interval where the modulus of the input reflection coefficient satisfies the constraint $|\Gamma_{\text{in}}(f)| < 0.025$.

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BANDWIDTH where |Gamma_in| < 0.025
Single-stub solution 1: BW = 10.926 MHz (2.359549 - 2.370475 GHz), FBW = 0.0046
Single-stub solution 2: BW = 3.858 MHz (2.363055 - 2.366913 GHz), FBW = 0.0016
Lumped solution 1: BW = 33.907 MHz (2.347986 - 2.381893 GHz), FBW = 0.0143
Lumped solution 2: BW = 33.910 MHz (2.348227 - 2.382137 GHz), FBW = 0.0143
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Figure 3: Frequency intervals where $|\Gamma_{\text{in}}| < 0.025$ for the single-stub and lumped matching solutions.

As shown in Figure 3, the lumped matching networks provide a wider frequency interval below the imposed threshold compared to the single-stub solutions.

4 Multisection maximum flat matching transformer

A multisection maximum flat (binomial) matching transformer is composed of N quarter-wavelength transmission line sections, each characterized by a different characteristic impedance Z_k . All sections have an electrical length $\theta = \pi/2$ at the design frequency f_0 .

In order to achieve a maximally flat behavior of the input reflection coefficient around f_0 , the characteristic impedances are selected according to a binomial distribution. Defining the coefficients

$$a_k = \frac{1}{2^N} \sum_{i=0}^{k-1} \binom{N}{i}, \quad k = 1, \dots, N, \quad (8)$$

the characteristic impedance of the k -th section is given by

$$Z_k = Z_0^{1-a_k} Z_L^{a_k}. \quad (9)$$

In the following, the theoretical solutions are reported for $Z_0 = 50 \Omega$ and $Z_L = 705 \Omega$.

Case $N = 3$

For $N = 3$, the binomial coefficients are $[1, 3, 3, 1]$, leading to the coefficients

$$a_1 = \frac{1}{8}, \quad a_2 = \frac{1}{2}, \quad a_3 = \frac{7}{8}.$$

The corresponding characteristic impedances are

$$Z_1 \approx 69.6 \Omega, \quad (10)$$

$$Z_2 = \sqrt{Z_0 Z_L} \approx 187.8 \Omega, \quad (11)$$

$$Z_3 \approx 506.4 \Omega. \quad (12)$$

Case $N = 4$

For $N = 4$, the binomial coefficients are $[1, 4, 6, 4, 1]$, leading to the coefficients

$$a_1 = \frac{1}{16}, \quad a_2 = \frac{5}{16}, \quad a_3 = \frac{11}{16}, \quad a_4 = \frac{15}{16}.$$

Therefore, the characteristic impedances of the four sections are

$$Z_1 \approx 59.0 \Omega, \tag{13}$$

$$Z_2 \approx 114.3 \Omega, \tag{14}$$

$$Z_3 \approx 308.4 \Omega, \tag{15}$$

$$Z_4 \approx 597.5 \Omega. \tag{16}$$

The obtained multisection transformers provide an exact impedance match at the design frequency f_0 and ensure a maximally flat response of the reflection coefficient in the vicinity of f_0 .

5 Multisection transformer frequency response

The frequency behavior of the multisection maximum flat matching transformer is analyzed in the range $0.3f_0 \leq f \leq 1.9f_0$. The modulus of the input reflection coefficient is evaluated for both $N = 3$ and $N = 4$ matching transformers by comparing the exact solution obtained using ABCD matrices with the approximated solution derived from the small reflection theory.

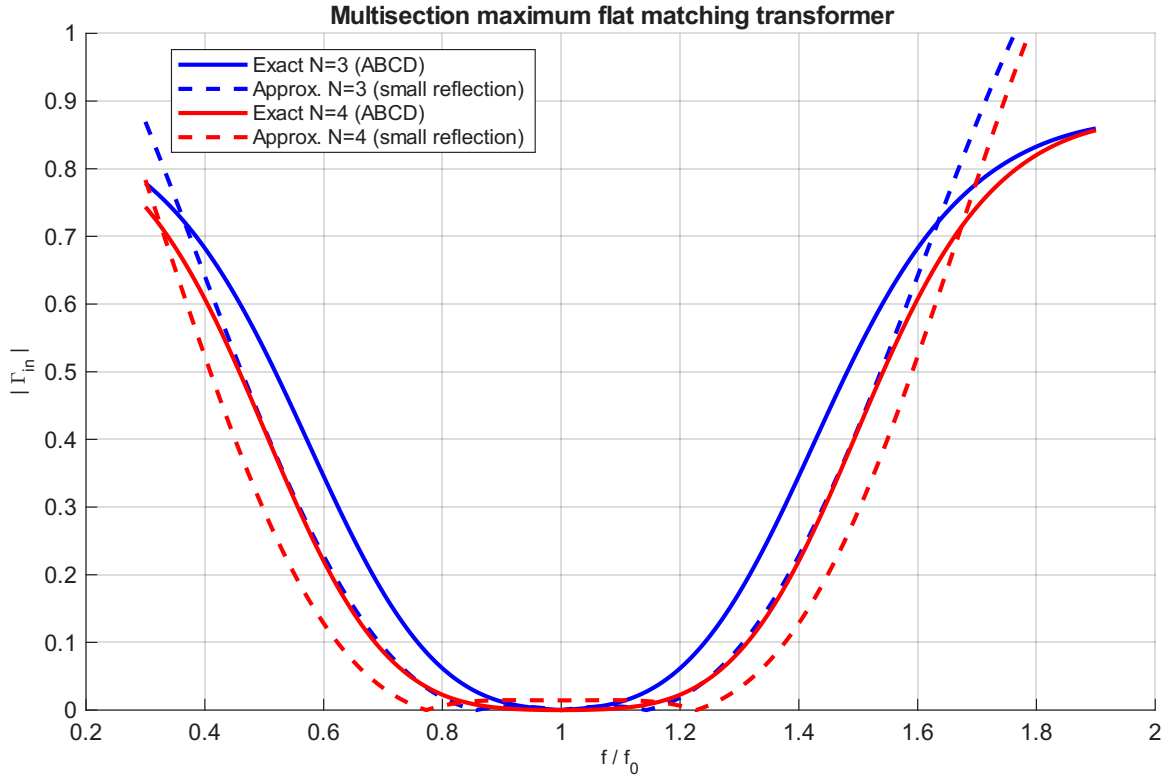


Figure 4: Modulus of the input reflection coefficient for multisection maximum flat matching transformers. Comparison between exact solutions obtained using ABCD matrices (continuous lines) and approximated solutions based on the small reflection theory (dashed lines) for $N = 3$ and $N = 4$.

6 Analytical bandwidth estimation

The bandwidth of the maximum-flat multisection matching transformer is analytically estimated using the small-reflection theory. The required condition is

$$|\Gamma| < 0.025. \quad (17)$$

For a binomial transformer with N quarter-wavelength sections, the input reflection coefficient can be approximated as

$$|\Gamma(\theta)| \approx \frac{|\ln(Z_L/Z_0)|}{2} |\cos \theta|^N, \quad \theta = \frac{\pi f}{2 f_0}. \quad (18)$$

Imposing the threshold $|\Gamma| < 0.025$, the resulting fractional bandwidth is

$$\frac{BW}{f_0} = \frac{4}{\pi} \arcsin \left[\left(\frac{2 \cdot 0.025}{|\ln(Z_L/Z_0)|} \right)^{1/N} \right]. \quad (19)$$

Using $Z_0 = 50 \Omega$ and $Z_L = 705 \Omega$ the following results are obtained:

- $N = 3$: $BW/f_0 \approx 0.343 \Rightarrow f/f_0 \in [0.828, 1.172]$.
- $N = 4$: $BW/f_0 \approx 0.484 \Rightarrow f/f_0 \in [0.758, 1.242]$.

As expected, increasing the number of sections leads to a wider bandwidth for the same reflection-coefficient constraint.

7 Exact bandwidth evaluation using ABCD matrices

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EXACT BANDWIDTH (ABCD), |Gamma| < 0.025
Maximum flat N=3:
  f1 = 2.038630 GHz, f2 = 2.691370 GHz
  BW = 652.740 MHz (BW/f0 = 0.2760)

Maximum flat N=4:
  f1 = 1.877621 GHz, f2 = 2.852379 GHz
  BW = 974.758 MHz (BW/f0 = 0.4122)

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Figure 5: Exact modulus of the input reflection coefficient obtained using ABCD matrices for maximum-flat multisection transformers with $N = 3$ and $N = 4$.

Based on the results obtained in point 5, the exact frequency bandwidth satisfying the condition $|\Gamma| < 0.025$ is evaluated by means of ABCD matrices for the maximum-flat multisection matching transformers with $N = 3$ and $N = 4$ sections. For the maximum-flat transformer with $N = 3$ sections, the condition $|\Gamma| < 0.025$ is satisfied in the frequency range

$$f \in [2.039, 2.691] \text{ GHz},$$

corresponding to a bandwidth of

$$\text{BW}_{N=3} = 652.7 \text{ MHz}, \quad \text{BW}_{N=3}/f_0 = 0.276.$$

For the maximum-flat transformer with $N = 4$ sections, the bandwidth increases to

$$f \in [1.878, 2.852] \text{ GHz}.$$

Therefore,

$$\text{BW}_{N=4} = 974.8 \text{ MHz}, \quad \text{BW}_{N=4}/f_0 = 0.412.$$

As expected, increasing the number of sections leads to a wider bandwidth while preserving the same reflection-coefficient constraint, confirming the superior matching performance of higher-order maximum-flat transformers.

8 Tolerance analysis for the $N = 4$ maximum flat transformer

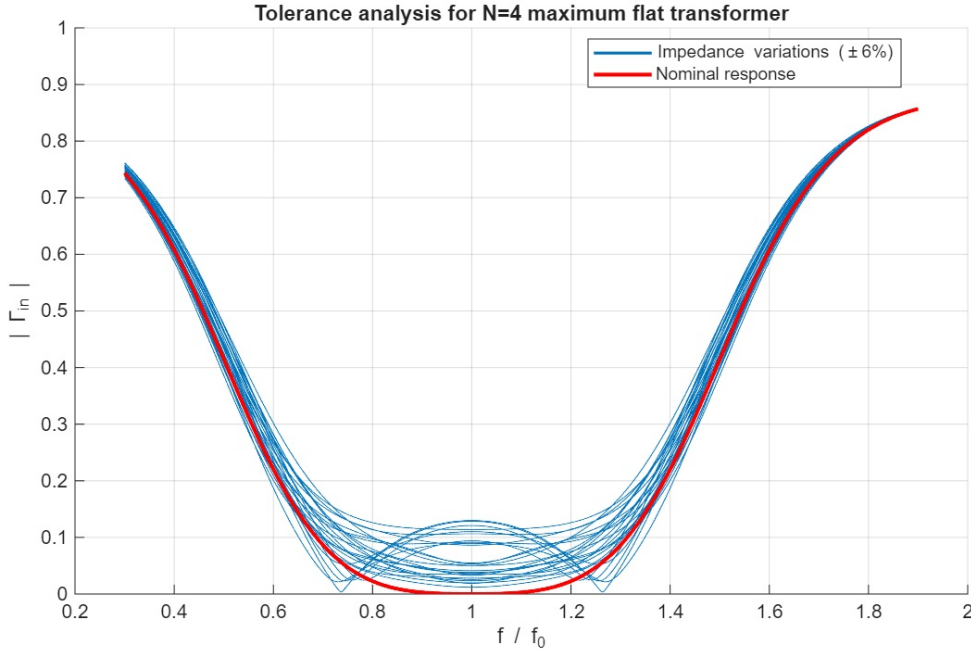


Figure 6: Tolerance analysis for the maximum-flat transformer with $N = 4$ sections: modulus of the input reflection coefficient for 25 realizations with random uniform impedance variations $\pm 6\%$ (blue curves) compared to the nominal response (red curve).

As expected, the dispersion is limited around the design frequency f_0 , whereas it increases as the frequency moves away from f_0 . Overall, the plot highlights the sensitivity of the matching performance to impedance tolerances and confirms that the best matching is achieved in the vicinity of f_0 .