

Microwave Devices - Homework n.3

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Considering my academic student number, the following parameters are used for this homework:

$$F = 5 \Omega, \quad f_0 = 1200 \text{ MHz}, \quad K = 5$$

The branch-line coupler used is shown in Fig. 1.

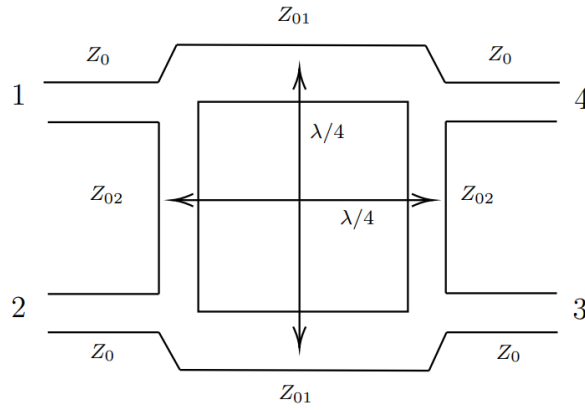


Figure 1: Schematic of a typical branch-line.

The characteristic impedance of the 4 ports is $Z_0 = 80 \Omega$.

1 Analytical solutions for Z_{01} and Z_{02}

Thanks to the previous considerations, and since the device is linear, we can write that (considering an excitation a_4 at port 4):

$$\begin{cases} b_1 = b_1^E + b_1^O = \frac{a_4}{2} T_E + \frac{a_4}{2} T_O \\ b_2 = b_2^E + b_2^O = \frac{a_4}{2} T_E - \frac{a_4}{2} T_O \\ b_3 = b_3^E + b_3^O = \frac{a_4}{2} \Gamma_E - \frac{a_4}{2} \Gamma_O \\ b_4 = b_4^E + b_4^O = \frac{a_4}{2} \Gamma_E + \frac{a_4}{2} \Gamma_O \end{cases} \quad (1)$$

The ABCD matrix of the branch-line coupler (for each mode) is:

$$\begin{bmatrix} \underline{A} & \underline{B} \\ \underline{C} & \underline{D} \end{bmatrix}_{E,O} = \begin{bmatrix} \mp \frac{z_{01}}{z_{02}} & jz_{01} \\ j \frac{1}{z_{01}} - j \frac{z_{01}}{z_{02}^2} & \mp \frac{z_{01}}{z_{02}} \end{bmatrix},$$

where the minus sign corresponds to the even mode and the plus sign to the odd mode. The quantities z_{01}

and z_{02} are the normalized impedances:

$$z_{01} = \frac{Z_{01}}{Z_0}, \quad z_{02} = \frac{Z_{02}}{Z_0}.$$

The matching condition at port 4 requires:

$$b_4 = 0 \quad \implies \quad \Gamma_E = -\Gamma_O.$$

The isolation condition for this exercise is:

$$S_{41} = 0 \quad \implies \quad b_1 = 0.$$

From the first equation in system (1), it follows that:

$$\frac{a_4}{2}(T_E + T_O) = 0 \quad \implies \quad T_E = -T_O.$$

Therefore the conditions for this device are:

$$\Gamma_E = -\Gamma_O, \quad T_E = -T_O.$$

Using the ABCD matrix, the even- and odd-mode transmission and reflection coefficients are:

$$T_{E,O} = \frac{2}{\underline{A}_{E,O} + \underline{B}_{E,O} + \underline{C}_{E,O} + \underline{D}_{E,O}}, \quad \Gamma_{E,O} = \frac{\underline{B}_{E,O} - \underline{C}_{E,O}}{\underline{A}_{E,O} + \underline{B}_{E,O} + \underline{C}_{E,O} + \underline{D}_{E,O}}.$$

By imposing the conditions $T_E = -T_O$ and $\Gamma_E = -\Gamma_O$ and substituting the expressions of $\underline{A}, \underline{B}, \underline{C}, \underline{D}$ for the even and odd case, we obtain the equation:

$$-\frac{z_{01}}{z_{02}} + jz_{01} + j\frac{1}{z_{01}} - j\frac{z_{01}}{z_{02}^2} = \frac{z_{01}}{z_{02}} - jz_{01} - j\frac{1}{z_{01}} + j\frac{z_{01}}{z_{02}^2}.$$

By simplifying we obtain:

$$2jz_{01} + \frac{2j}{z_{01}} - 2j\frac{z_{01}}{z_{02}^2} = 0 \quad \implies \quad z_{01} + \frac{1}{z_{01}} = \frac{z_{01}}{z_{02}^2}.$$

Rearranging:

$$z_{01}^2 + 1 = \frac{z_{01}^2}{z_{02}^2} \quad \implies \quad z_{02} = \frac{z_{01}}{\sqrt{z_{01}^2 + 1}}. \quad (2)$$

Thanks to eq. (2), by exploiting the relation between z_{01} and z_{02} , the ABCD matrix of the equivalent two-port (for both even and odd mode) can be written in normalized form as

$$\begin{pmatrix} \underline{A} & \underline{B} \\ \underline{C} & \underline{D} \end{pmatrix}_{E,O} = \begin{pmatrix} \mp\sqrt{1+z_{01}^2} & jz_{01} \\ -jz_{01} & \mp\sqrt{1+z_{01}^2} \end{pmatrix},$$

where the minus sign refers to the even mode and the plus sign to the odd mode.

Starting from the ABCD matrix, we can compute the transmission and reflection coefficients as:

$$T = \frac{2}{\underline{A} + \underline{B} + \underline{C} + \underline{D}}, \quad \Gamma = \frac{\underline{B} - \underline{C}}{\underline{A} + \underline{B} + \underline{C} + \underline{D}}.$$

Applying these formulas to the even and odd cases, we obtain:

$$T_E = \frac{2}{\underline{A}_E + \underline{B}_E + \underline{C}_E + \underline{D}_E} = -\frac{1}{\sqrt{z_{01}^2 + 1}}, \quad (3)$$

$$T_O = \frac{2}{\underline{A}_O + \underline{B}_O + \underline{C}_O + \underline{D}_O} = +\frac{1}{\sqrt{z_{01}^2 + 1}}, \quad (4)$$

$$\Gamma_E = \frac{\underline{B}_E - \underline{C}_E}{\underline{A}_E + \underline{B}_E + \underline{C}_E + \underline{D}_E} = -\frac{jz_{01}}{\sqrt{z_{01}^2 + 1}}, \quad (5)$$

$$\Gamma_O = \frac{\underline{B}_O - \underline{C}_O}{\underline{A}_O + \underline{B}_O + \underline{C}_O + \underline{D}_O} = +\frac{jz_{01}}{\sqrt{z_{01}^2 + 1}}. \quad (6)$$

The calculation of these coefficients allows us to solve system (1), obtaining:

$$\begin{cases} b_1 = \frac{a_4}{2} (T_E + T_O) = 0, \\ b_2 = \frac{a_4}{2} (T_E - T_O) = -\frac{a_4}{\sqrt{z_{01}^2 + 1}}, \\ b_3 = \frac{a_4}{2} (\Gamma_E - \Gamma_O) = -\frac{jz_{01}a_4}{\sqrt{z_{01}^2 + 1}}, \\ b_4 = \frac{a_4}{2} (\Gamma_E + \Gamma_O) = 0. \end{cases} \quad (7)$$

The results are in agreement with the initial conditions: port 4 is matched ($b_4 = 0$) and ports 4 and 1 are decoupled ($b_1 = 0$). By exploiting the symmetry of the branch-line coupler, the complete scattering matrix can finally be written as

$$S = \begin{pmatrix} 0 & -\frac{jz_{01}}{\sqrt{z_{01}^2 + 1}} & \frac{1}{\sqrt{z_{01}^2 + 1}} & 0 \\ -\frac{jz_{01}}{\sqrt{z_{01}^2 + 1}} & 0 & 0 & \frac{1}{\sqrt{z_{01}^2 + 1}} \\ \frac{1}{\sqrt{z_{01}^2 + 1}} & 0 & 0 & -\frac{jz_{01}}{\sqrt{z_{01}^2 + 1}} \\ 0 & \frac{1}{\sqrt{z_{01}^2 + 1}} & -\frac{jz_{01}}{\sqrt{z_{01}^2 + 1}} & 0 \end{pmatrix}. \quad (8)$$

The scattering matrix allows us to perform some checks about the correctness of the obtained results: this device is confirmed to be lossless, symmetrical and reciprocal. From matrix (8) we can extract the element S_{42} , which must be made equal to $k \cdot 0.1$. In my case $k = 5$, so in order to calculate the real value of the normalized characteristic impedance z_{01} we have to impose:

$$|S_{42}| = k \cdot 0.1 = 0.5 = \frac{1}{\sqrt{z_{01}^2 + 1}} \implies z_{01} = \sqrt{3}. \quad (9)$$

Using this value for z_{01} and eq. (2) to calculate z_{02} , we obtain $|S_{42}| = 0.5$ and $S_{44} = 0$.

2 Calculation of w and h with Hammerstad approximations

In this homework the substrate parameters are not fixed a priori: we are asked to choose the relative permittivity and the substrate height in the ranges $3.1 \leq \varepsilon_r \leq 3.9$ and $2.4 \text{ mm} \leq h \leq 3.6 \text{ mm}$. In order to have realistic values, in the following we select

$$\varepsilon_r = 3.5, \quad h = 3 \text{ mm}.$$

The equation considered to calculate the ratio w/h (when the latter is bigger than 2) is the following:

$$\frac{w}{h} = \frac{2}{\pi} \left\{ B - 1 - \ln(2B - 1) + \frac{\varepsilon_r - 1}{2\varepsilon_r} \left[\ln(B - 1) + 0.39 - \frac{0.61}{\varepsilon_r} \right] \right\}, \quad B = \frac{120\pi^2}{2Z_x\sqrt{\varepsilon_r}} \quad (10)$$

While if the ratio $w/h < 2$ then the equation to be used is:

$$\frac{w}{h} = \frac{8}{e^A - 2e^{-A}}, \quad A = \frac{Z_x}{60} \sqrt{\frac{\varepsilon_r + 1}{2}} + \frac{\varepsilon_r - 1}{\varepsilon_r + 1} \left(0.23 + \frac{0.11}{\varepsilon_r} \right), \quad (11)$$

considering $Z_x = Z_{01}$ or Z_{02} . By calculating eq. 10 for Z_{01} we have:

$$\frac{w}{h} = 0.217, \quad \Rightarrow \quad w_{01} = 0.65 \text{ mm.}$$

While for Z_{02} we must use eq. 11, as the ratio is lower than 2, obtaining:

$$\frac{w}{h} = 1.28, \quad \Rightarrow \quad w_{02} = 3.85 \text{ mm.} \quad (12)$$

By using eq. 13 we can also calculate the effective permittivity of the two lines, which are:

$$\varepsilon_{\text{eff}} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2} \left(\frac{1}{\sqrt{1 + 12 h/w}} \right), \quad (13)$$

from which we obtain:

$$\varepsilon_{\text{eff},01} = 2.42, \quad \varepsilon_{\text{eff},02} = 2.64.$$

3 Scattering coefficients' plots

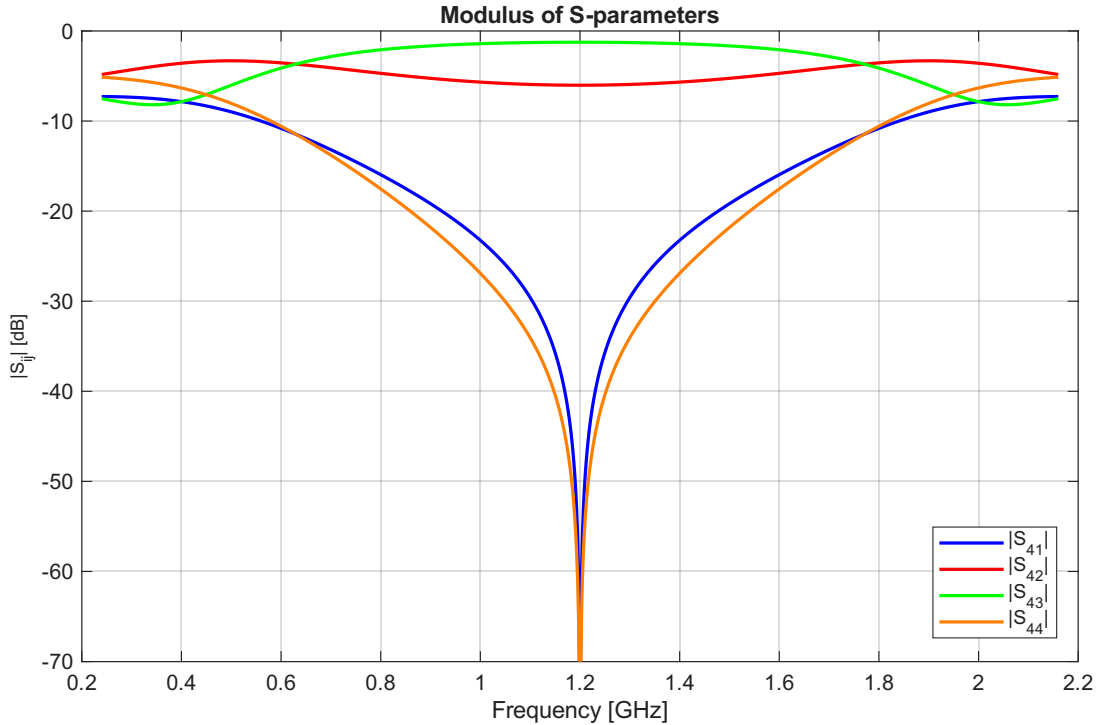


Figure 2: Plots of the modulus (in dB) of the scattering coefficients S_{41} , S_{42} , S_{43} and S_{44} in the range $0.1f_0 \leq f \leq 1.9f_0$.

It has to be noted that $|S_{41}|$ and $|S_{44}|$ follow a very similar trend, both vanishing at the central frequency.

From the plot of $|S_{44}|$ we see that, close to the central frequency f_0 , the curve reaches very low values (numerically below -60 dB), which means that port 4 is well matched, as required by the design condition $S_{44} = 0$ at f_0 .

We can also notice that $|S_{42}|$ is approximately equal to the prescribed coupling factor $k \cdot 0.1 = 0.5$ around f_0 , which in dB corresponds to

$$20 \log_{10}(0.5) \simeq -6.02 \text{ dB},$$

while $|S_{43}| = \sqrt{3}/2 \simeq -1.25$ dB, so that $|S_{42}|^2 + |S_{43}|^2 = 1$. In conclusion, the trend of $|S_{41}|$ shows that port 4 and port 1 are effectively decoupled at the central frequency, in agreement with the theoretical conditions derived in point 1.

4 Scattering coefficients plots considering a $\pm 5\%$ random variation to Z_{01} and Z_{02}

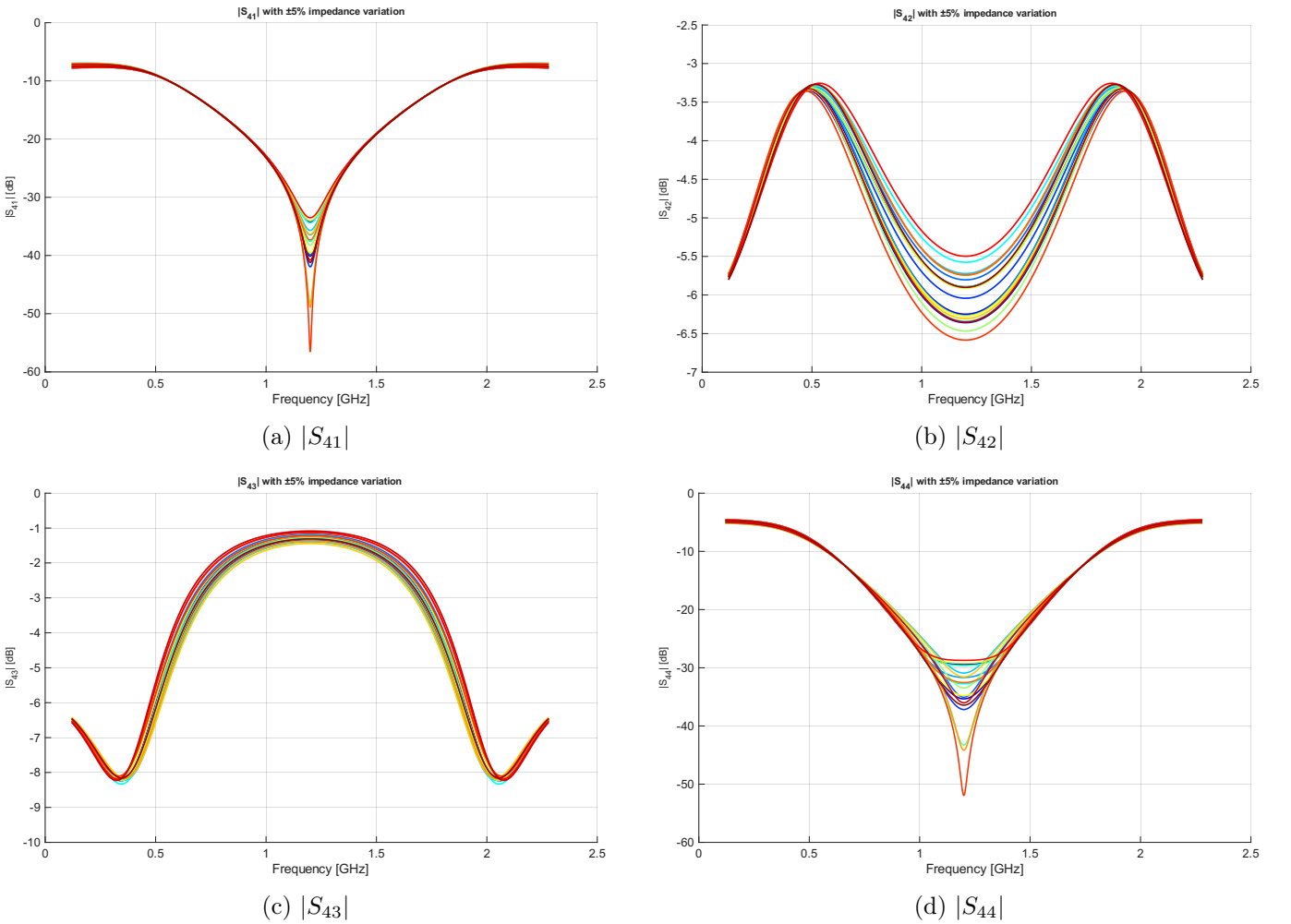


Figure 3: Analysis with $\pm 5\%$ random variation of Z_{01} and Z_{02} .

In fig. 3 the scattering coefficients S_{41} , S_{42} , S_{43} and S_{44} are shown when a $\pm 5\%$ random variation is applied to the nominal values of Z_{01} and Z_{02} calculated in the previous sections. The frequency range considered is still $0.1f_0 \leq f \leq 1.9f_0$ and the number of generated samples is 20. From the plots it can be observed that the coupling and isolation curves are slightly spread around the nominal trend, confirming the sensitivity of the device to variations in the characteristic impedances.

5 CST plots

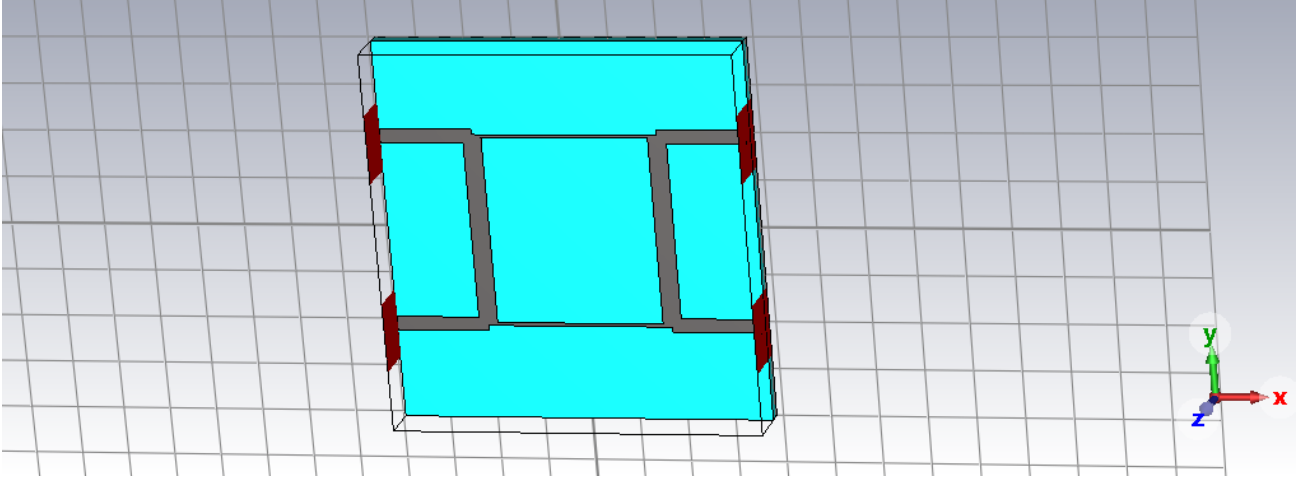


Figure 4: Model of the branch-line coupler implemented in CST.

In Fig. 4 the CST model of the branch-line coupler is shown.

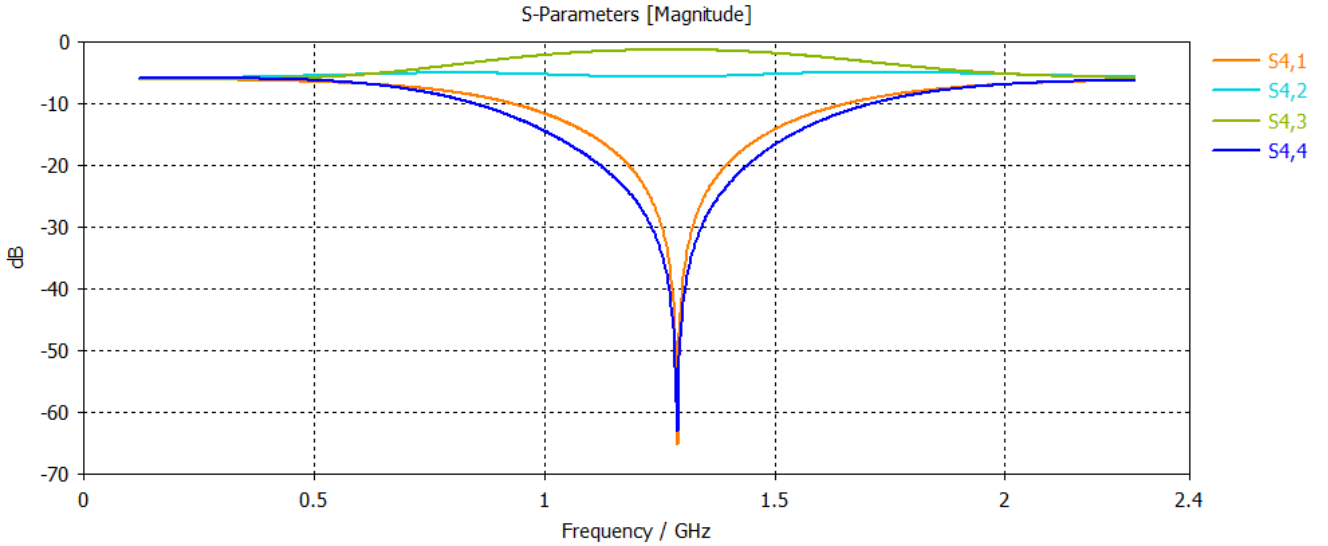


Figure 5: Magnitude (in dB) of the scattering parameters S_{41} , S_{42} , S_{43} and S_{44} in the frequency range $0.1f_0 \leq f \leq 1.9f_0$, obtained from CST simulations.

From Fig. 5 it can be observed that the operating frequency of the simulated branch-line coupler is not exactly centered at the theoretical value $f_0 = 1.2$ GHz, but it is slightly shifted toward higher frequencies. This discrepancy is mainly due to the fact that the analytical Matlab model assumes ideal transmission lines and neglects the effect of discontinuities caused by changes in microstrip width, while CST fully accounts for parasitic effects at the junctions between lines with different characteristic impedances (Z_{01} and Z_{02}). In particular, capacitive and inductive effects at the corners of the structure lead to a deviation from the ideal behavior and produce the frequency shift observed in the simulated response.