

Microwave Devices - Homework n.1

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Considering my academic student number, the following parameters are used for this homework: $a = 0.0610$ m, $b = 0.0218$ m, $\varepsilon_r = 3.9$, $m = 2$, $n = 4$, Mode = TE_{2,4}.

1 Cut-off frequencies

Table 1 reports the cut-off frequencies of all the propagating modes, ordered by increasing cut-off frequency.

Table 1: Cutoff frequencies for different TE_{mn} modes.

m	n	f_c [GHz]	m	n	f_c [GHz]
1	0	1,245	6	2	10,209
2	0	2,489	0	3	10,455
0	1	3,481	1	3	10,519
1	1	3,697	8	1	10,546
3	0	3,734	2	3	10,738
2	1	4,280	3	3	11,092
4	0	4,978	7	2	11,152
3	1	5,105	9	0	11,201
4	1	6,074	4	3	11,371
5	0	6,223	9	1	11,728
0	2	6,964	8	2	12,148
1	2	7,074	5	3	12,158
5	1	7,130	10	0	12,446
2	2	7,395	6	3	12,839
6	0	7,468	10	1	12,921
3	2	7,901	9	2	13,187
6	1	8,238	7	3	13,600
4	2	8,559	11	0	13,691
7	0	8,712	0	4	13,927
5	2	9,338	1	4	13,983
7	1	9,380	11	1	14,123
8	0	9,957	2	4	14,148

While TE_{mn} modes may exist even when $m = 0$ or $n = 0$, TM_{mn} modes require both indices to be non-zero. Therefore, TM modes are only possible for $m \geq 1$ and $n \geq 1$.

2 Components of the electromagnetic field

Starting from the longitudinal component of the magnetic field:

$$H_z(x, y, z) = H_0 \cos\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-j\beta z}$$

where

$$\beta = \sqrt{k^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}, \quad k = \omega\sqrt{\mu\varepsilon}.$$

The cutoff frequency is:

$$f_c = \frac{1}{2\pi\sqrt{\mu\varepsilon}} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}.$$

From Maxwell's equations, the transverse components of the fields can be derived as follows.

Electric field components:

$$E_x = -j \frac{\omega\mu}{k_c^2} \frac{\partial H_z}{\partial y} = j \frac{\omega\mu n\pi}{k_c^2 b} H_0 \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-j\beta z}$$

$$E_y = j \frac{\omega\mu}{k_c^2} \frac{\partial H_z}{\partial x} = -j \frac{\omega\mu m\pi}{k_c^2 a} H_0 \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-j\beta z}$$

$$E_z = 0$$

Magnetic field components:

$$H_x = -j \frac{\beta}{k_c^2} \frac{\partial H_z}{\partial x} = j \frac{\beta m\pi}{k_c^2 a} H_0 \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-j\beta z}$$

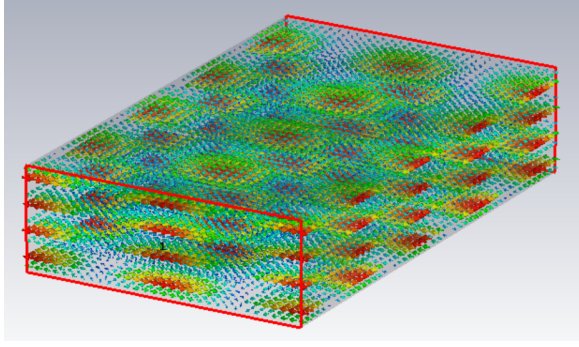
$$H_y = -j \frac{\beta}{k_c^2} \frac{\partial H_z}{\partial y} = j \frac{\beta n\pi}{k_c^2 b} H_0 \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-j\beta z}$$

where

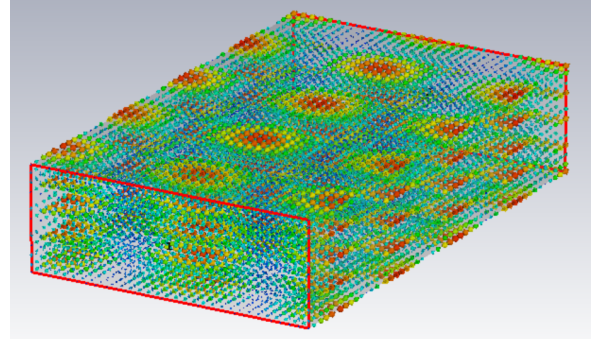
$$k_c^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2.$$

3 Analysis using CST

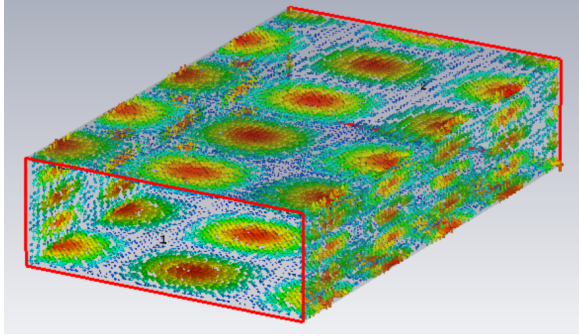
The next plots illustrate the spatial distribution of the electric and magnetic fields, the lines of the electric currents on the walls and the active power of the given mode.



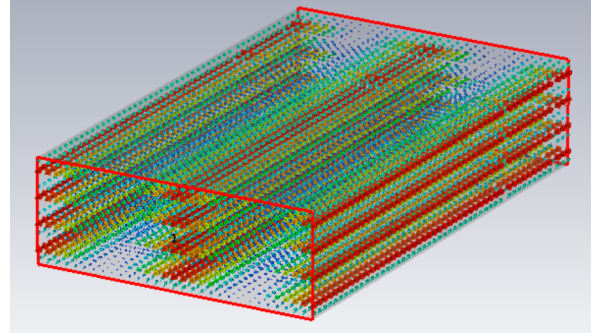
(a) Electric field.



(b) Magnetic field.



(c) Surface current on the walls.



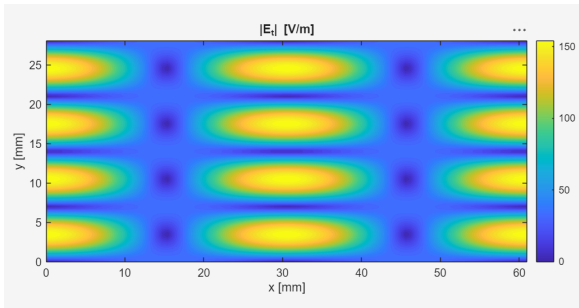
(d) Active power flow along the propagation direction.

Figure 1: Field and power distributions for TE_{24} mode at $f = 14.09$ GHz.

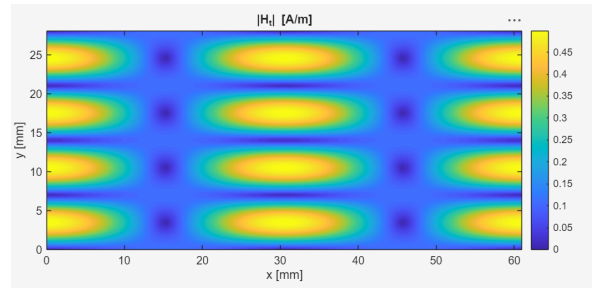
The CST results fit well with the analytical solutions from points 1 and 2, validating both the cutoff frequency and the modal field distribution.

4 Comparison between CST and MATLAB

The following plots demonstrate that the results obtained from CST are consistent with the MATLAB results.



(a) Transverse electric field magnitude $|E_t|$.



(b) Transverse magnetic field magnitude $|H_t|$.

Figure 2: MATLAB transverse field magnitudes for $TE_{2,4}$ mode.

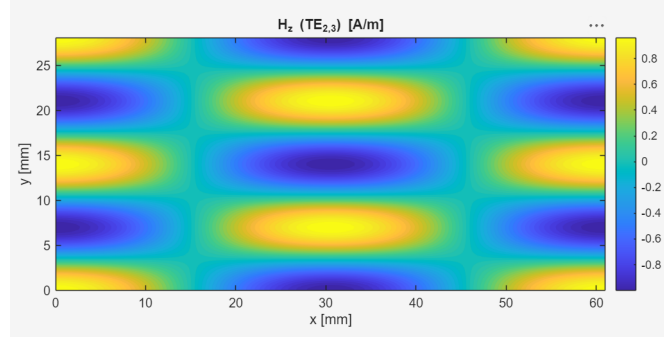
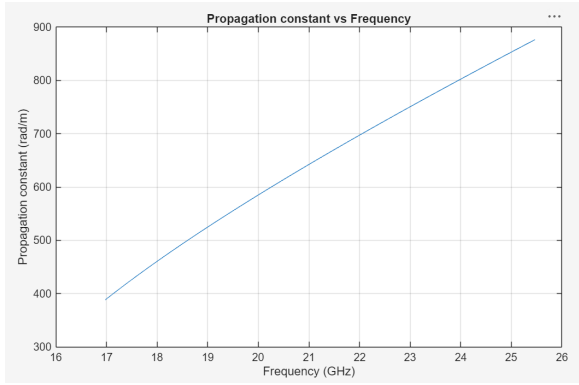


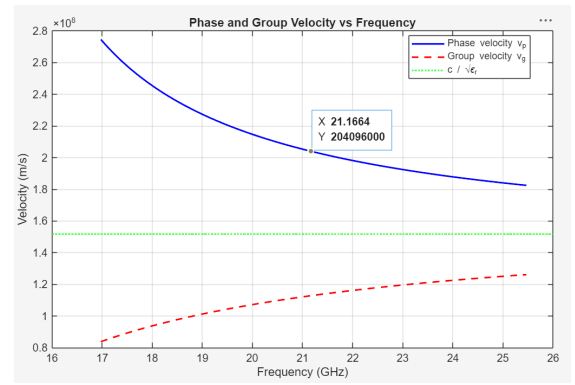
Figure 3: Longitudinal magnetic field H_z for $TE_{2,4}$ mode.

5 Propagation characteristics vs frequency

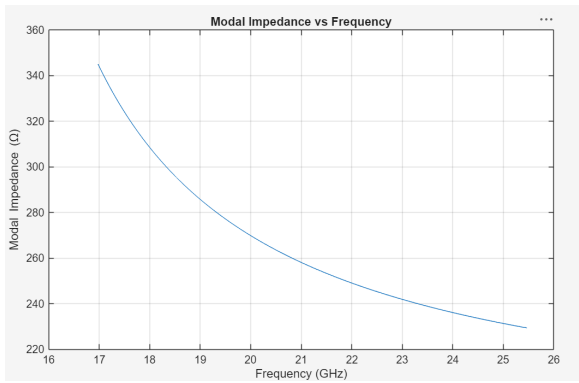
For the following plots, the propagation parameters of the TE_{24} mode are shown as a function of frequency in the range $1.2f_c < f < 1.8f_c$.



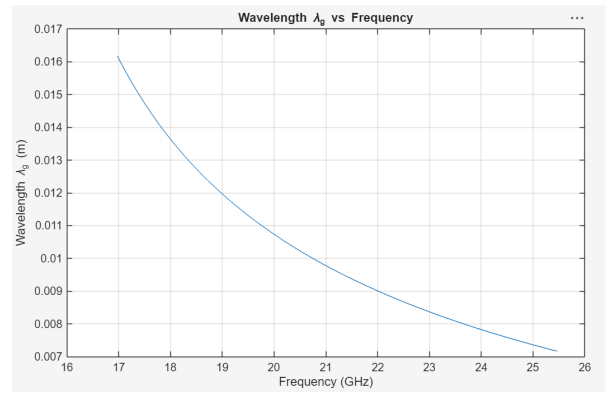
(a) Propagation constant β vs frequency.



(b) Phase and group velocities v_f and v_g vs frequency.



(c) Modal impedance Z_{TE} vs frequency.



(d) Wavelength inside the guide λ_g vs frequency.

Figure 4: Propagation parameters of the TE_{24} mode as functions of frequency.

6 Differential delay with respect to the fastest cut-off mode

The plot below shows how the accumulated time delay over 90 meters of waveguide decreases with frequency. As the frequency increases, both modes tend to propagate closer to the speed $c/\sqrt{\epsilon_r}$, as it can be observed the delay difference is reduced.

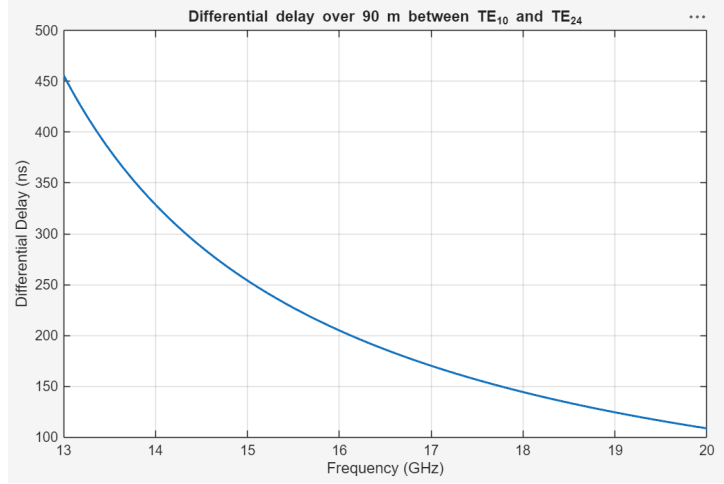


Figure 5: Differential delay over 90 m between TE_{10} and TE_{24} modes.

7 Differential delay with respect to the closest cut-off mode

For this analysis, the differential delay over 90 meters of waveguide has been computed between the given mode TE_{24} and the mode with the closest cut-off frequency, $TE_{11,1}$.

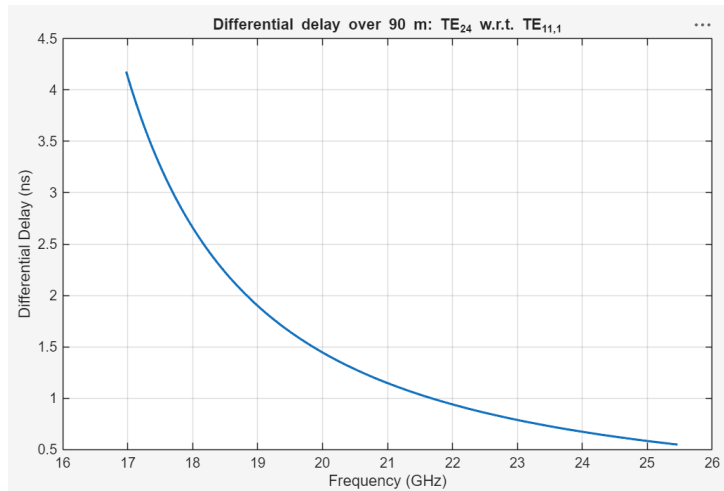


Figure 6: Differential delay over 90 m between TE_{24} and $TE_{11,1}$ modes.

As can be observed, the differential delay decreases with frequency, similar to the behaviour shown in Figure 5. Since the two cut-off frequencies are very close, unlike the previous case, the delay difference remains very small.

8 Modal filter

To avoid suppressing the $TE_{2,4}$ mode, the electric wall was placed perpendicular to the electric field direction, so that the tangential component of E is zero for the desired mode. Instead, regarding the thin slot, it has been placed parallel to the surface current direction, in order to not disturb the $TE_{2,4}$ mode while attenuating other propagating modes.

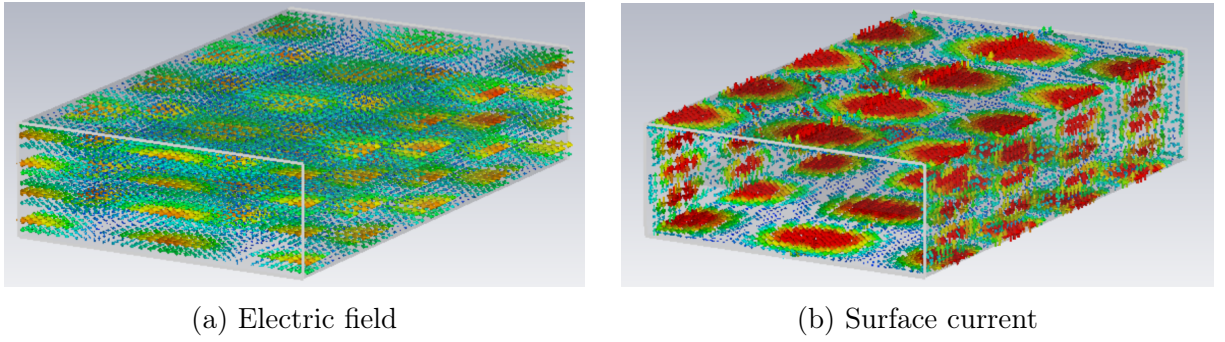


Figure 7: Both figures represent the $TE_{2,4}$ mode with the electric wall and the thin slot included.

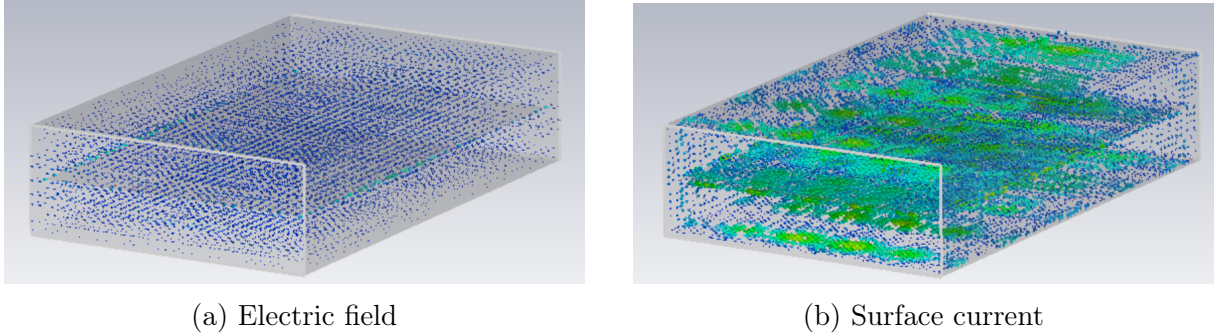


Figure 8: Field distribution of $TE_{0,1}$ mode, an unwanted mode after inserting the electric wall and thin slot. The mode is visibly attenuated and does not propagate through the waveguide as we expected.

The modes that may still propagate in addition to the $TE_{2,4}$ mode are:

$$\begin{aligned}
 &TE_{1,0}, TE_{2,0}, TE_{3,0}, TE_{4,0}, TE_{5,0}, TE_{6,0}, TE_{7,0}, TE_{8,0}, TE_{9,0}, TE_{11,0}, \\
 &TE_{0,2}, TE/TM_{1,2}, TE/TM_{2,2}, TE/TM_{3,2}, TE/TM_{4,2}, TE/TM_{5,2}, TE/TM_{6,2}, \\
 &TE/TM_{7,2}, TE/TM_{9,2}, TE_{0,4}, TE/TM_{1,4}, TE/TM_{2,4}.
 \end{aligned}$$